# Evaluation of Different Tariff Rate offered by Mobile Telephone Operators Using Personal Social Network Model 

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#### Abstract

Different tariff offer of the mobile telecommunication operator Grameenphone (Telenor Group) is evaluated using personal social network model. It is found that the offer of price is valid and provides some consumer surplus also. An optimum point of call rate is found beyond which consumer surplus becomes negative. So this personal social tie network model can be used for evaluation as well as for optimization of any tariff offer.


Index Terms- Social tie, Tariff plans, Pricing policy, Consumer Surplus, Telecommunication Market, Utility, Network Externality.

## 1 Introduction

Network externality or Network effect is a very important parameter in telecommunications market. When consuming a network good, consumers' value is not only from quality and quantity of certain products, but also from the size of the network of the product, which means the number of users in the network increase, the value of the network to other users also changes [1]. In case of telecommunications market the subscriber will use the telecommunication service to communication with each other and consumers will value more of a certain network if more number of people in that telecommunications network.

The cheaper the tariff rate or the price of the service, the more communication will be created and the more value of consumer will be produced. The closer the relation between the communicator, the more demand for the telecommunication service will generate. Therefore, the lower tariff rate or the price of the service between the communicator with close tie or relationship will produce large consumer value and surplus.

If we shift our focus from consumption of network to consumption of individual communication through the network, the personal social network's impact on consumer's value and demand becomes very important [1]. For example, a mobile phone subscriber evaluates his certain call with someone else on the basis of the relationship with this personal he talks with through mobile network. This user's utility would be high if talking with closed friends or family members, while his utility would be minimized when he receives a call from a sales agent who has weak tie with him. Thus we can say that the marginal utility obtained by this subscriber from strong tie communication is bigger than marginal utility from weak tie communication. Therefore the subscriber's demand for the communication with someone with strong tie with this subscriber is higher than the demand for communication with weak-tie person.

Let us consider a telecommunications market that consists of N subscribers who demand for the communication service. The value of the communications between a pair of subscriber depends on the strength of their social tie.

We represent strength of tie between two subscribers with a one-dimensional index variable, denoted by t . Consistent with literature on social network analysis, we assume three levels of tie strength: strong ties (denoted by $t=s)$, weak ties $(t=w)$, and absence of tie $(\mathrm{t}=0)$. [2] [3] [4]

A subscriber will obtain positive utility from the communication with another subscriber if and only if these two subscribers have either a strong $(\mathrm{t}=\mathrm{s})$ or weak $(\mathrm{t}=\mathrm{w})$ tie. Communication between a pair of subscriber with no social tie ( $t=0$ ) will produce zero utility or negative utility. Let us define these subscribers with whom a subscriber has either a strong or a weak tie as the consumer's personal communication network, where $t \in\{s, w\}$.

## 2 Discriminatory Pricing Policy Based on Social TIE

The telecommunications service providers follow discriminatory pricing scheme by offering different price plan for different social tie network. Let us define a price plan $\left(p_{s}, p_{w}\right)$, where $p_{s}$ for the communications between subscribers with strong ties and $p_{w}$ for the communications between subscribers with weak tie. Amount of service consumption can be measured either in minutes or seconds.

But the numbers of strong tie within the consumer's personal communication network is limited by the telecommunications service providers'. The numbers of strong tie within the consumer's personal communication network, which we call the numbers of Friends and Family (FnF) member, vary with different telecommunications service providers'. The variable fees of the price plan generally depend on strength of ties. We call the difference between two variable fees tie strength-based discount. This discount, in another word, is the consumer surplus.

## 3 Utility and Consumer Surplus in Telecommunications Market

A consumer's utility from communications with another person depends on strength of their tie. For a pair of subscribers with a tie
strength t , we let $u_{t}(q)$ denotes each subscriber's utility from their communications of amount $q$ and propose a quadratic utility function as follows:

$$
\begin{equation*}
u_{t}\left(q_{t}\right)=\alpha_{1}^{t} q_{t}-\frac{\alpha_{2}^{t}}{2} q_{t}^{2}, \text { where } \alpha_{1}^{t}>0, \alpha_{2}^{t}>0 \text { and } t \in\{s, w\} \tag{1}
\end{equation*}
$$ [5]

The values of the social tie strength coefficients $\alpha_{1}^{t}$ and $\alpha_{2}^{t}$ depend on the strength of social tie $(\mathrm{t})$. The concave utility function in equation (1) implies a decreasing marginal utility when amount of communication increases. As people tend to transmit more important information at the beginning of their conversations, such utility function fit well in the telecommunications market analysis. We also assume the absence of income effect because consumers' communication expense typically accounts for a small proportion of their budget.

Now to find out the optimal amount of communication of a subscriber with another subscriber we need to solve the consumer utility maximization problem with respect to $q$.
$q_{t}\left(p_{t}\right)=\arg \max \left\{u_{t}\left(q_{t}\right)-p_{t} q_{t}\right\}$
Here $u_{t}\left(q_{t}\right)$ is consumer utility function and $t \in\{s, w\}$.
When the optimal quantity of communication is positive, we could get an optimal quantity of communication through solving first-order condition of equation (2) i.e.:
$\frac{d}{d q}\left\{u_{t}\left(q_{t}\right)-p_{t} q_{t}\right\}=0$
Putting utility function from equation (1) into equation (3) and solving the equation (3) we get as follows:
$p_{t}=\alpha_{1}^{t}-\alpha_{2}^{t} q_{t}$
Equation (4) indicates that, for two subscribers that have a tie of strength of $t$, the marginal value of their communications is $\alpha_{1}^{t}$ for the first unit of consumption and then decreases by $\alpha_{2}^{t}$ for each additional unit of consumption.

Equation (4) is called the "Inverse Demand Function". We know quantity demanded $(\mathrm{Q})$ is a function of price $(\mathrm{P})$. The inverse demand function treats price as a function of quantity demanded, i.e. $P=f^{-1}(Q)$. It is also called the "Price Function".

Rearranging equation (4) we get the optimal demand:
$q_{t}=\frac{\alpha_{1}^{t}-p_{t}}{\alpha_{2}^{t}}$
Let $v_{t}\left(p_{t}\right)$ denotes the consumer surplus of a subscriber from the consumption of the communications service of amount $q_{t}$.

Substituting $q_{t}$ from equation (5) into the utility function (equation 1) we obtain the equation for the consumer surplus:

$$
\begin{equation*}
v_{t}\left(p_{t}\right)=\frac{1}{2 \alpha_{2}^{t}}\left(\alpha_{1}^{t}-p_{t}\right)^{2} \tag{6}
\end{equation*}
$$

## 4 Social Tie Strength Coefficient Determination

In this following section we have evaluated the social tie strength coefficients $\alpha_{1}^{t}$ and $\alpha_{2}^{t}$ under some certain conditions using equation (6). We have chosen the most popular mobile telephone operator in Bangladesh "Grameenphone" for our research work and taken their tariff packages as reference price plan. For simplicity we have only taken the tariff rates-
$\checkmark$ at "peak" hour
$\checkmark$ between the same telecom operator
$\checkmark \quad$ between strong tie network (i.e. within FnF member) and
$\checkmark$ without VAT and call establishment or connection charges
Considering only strong tie network i.e. in our real case scenario only within FnF members, equation (6) becomes:

$$
\begin{equation*}
v_{s}\left(p_{s}\right)=\frac{1}{2 \alpha_{2}^{s}}\left(\alpha_{1}^{s}-p_{s}\right)^{2} \tag{7}
\end{equation*}
$$

Let us consider two tariff packages of the mobile telephone operator "Grameenphone":

- Smile
- Baadhon

Now in "Smile" package:
Normal Grameenphone to Grameenphone tariff rate is $=1.50$ taka/min

FnF $(3 \mathrm{FnF})$ tariff rate, $p_{s}=0.49$ taka/min [6]
So for strong tie network consumer get a surplus, $v_{s}\left(p_{s}\right)=(1.50-$ 0.49) $=1.01 \mathrm{taka} / \mathrm{min}$

Thus for "Smile" package equation (7) becomes:
$1.01=\frac{1}{2 \alpha_{2}^{s}}\left(\alpha_{1}^{s}-0.49\right)^{2}$
$>2.02 \alpha_{2}^{s}=\alpha_{1}^{s^{2}}-0.98 \alpha_{1}^{s}+0.2401$
Now in "Baadhon" package:
If a subscriber migrate his package from "Smile" to "Baadhon" then he have to talk to his strong tie network at a flat rate of 0.79 taka/min, as "Baadhon" package has no different price plan for FnF. But the subscriber can still enjoy talking to his strong tie network at a lower rate than Smile which creates a surplus.

Tariff rate, $p_{s}=0.79$ taka $/ \mathrm{min}[7]$
So consumer surplus, $v_{s}\left(p_{s}\right)=(1.50-0.79)=0.71 \mathrm{taka} / \mathrm{min}$
Thus for "Baadhon" package equation (7) becomes:
$0.71=\frac{1}{2 \alpha_{2}^{s}}\left(\alpha_{1}^{s}-0.79\right)^{2}$
$\gg 1.42 \alpha_{2}^{s}=\alpha_{1}^{s^{2}}-1.58 \alpha_{1}^{s}+0.6241$
Now solving equation (8) and (9) we get:
$0.6 \alpha_{1}^{s 2}-1.8 \alpha_{1}^{s}+0.91974=0$
Using standard solution system of a quadratic equation we get from equation (10):

## $\alpha_{1}^{s}=2.34682$ or 0.65318

Now putting the values of $\alpha_{1}^{s}$ in equation (8) we get:
$\alpha_{2}^{s}=1.70676$ or 0.01318
So have two sets of solutions:
Solution Set 1: $\left(\alpha_{1}^{s}, \alpha_{2}^{s}\right)=(2.34682,1.70676)$
Solution Set 2: $\left(\alpha_{1}^{s}, \alpha_{2}^{s}\right)=(0.65318,0.01318)$

## 5 Simulation

In this following section will use the solution sets, obtained in the previous section, into equation (7) to generate some graphs (plot of tariff rate vs. consumer surplus) using MATLAB. We will observe the variation of consumer surplus in accordance with the tariff rate.

Using Solution Set 1 equation (7) becomes:
$v_{s}\left(p_{s}\right)=\frac{1}{3.41352}\left(2.34682-p_{s}\right)^{2}$

$$
\begin{equation*}
\text { Using Solution Set } 2 \text { equation (5.7) becomes: } \tag{11}
\end{equation*}
$$

$v_{s}\left(p_{s}\right)=\frac{1}{0.02636}\left(0.65318-p_{s}\right)^{2}$
Now we will use equation (11) and (12) to generate our desired plot.

Using Solution Set 1 the plot is like bellow:


Fig 1: Variation of consumer surplus with tariff rate (for Solution Set 1)
Using Solution Set 2 the plot is like bellow:


Fig 2: Variation of consumer surplus with tariff rate (for Solution Set 2)
sumer surplus cannot be more than offered price. Thus solution pair 2 is not acceptable.

But both the figures 1 and 2 shows that the subscribers of the mobile telephone operator enjoy an increasing consumer surplus as the call rate decreases. This result fits very much from both the point of views of standard economics and our general perception. Thus the price plan is justified from the point of view of our personal social network model.

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## 6 Result and Discussion

When the tariff rate is zero then the maximum consumer surplus that a subscriber can gain should be equal to the highest offered tariff rate. From figure 1 we see that at zero tariff rate the consumer surplus is approximately equal to $1.5 \mathrm{Tk} / \mathrm{min}$ which is the highest offered tariff rate of the two price plans we considered. At tariff rate $2.37 \mathrm{Tk} / \mathrm{min}$ the consumer surplus is zero. This is the optimum tariff rate that this mobile telephone operator can charge for these two tariff plans. Beyond the optimum tariff rate the consumer surplus is actually negative, though it appears to be positive. To explain the situation better that any price more than the amount that makes the consumer surplus zero will make consumer surplus negative. That means the tariff rate will become beyond consumer's expectation.

The company offers a tariff rate of $1.5 \mathrm{Tk} / \mathrm{min}$ and subscribers get some amount of consumer surplus. The company does it deliberately to increase the amount of communication by the subscribers that eventually increase the revenue of the company. The company always offers a tariff rate lower than the hardest rate (rate with zero consumer surplus). This operator can charge tariff rate up to 2.37 $\mathrm{Tk} / \mathrm{min}$. But how much lower rate it will offer depends on the detailed financial analysis of the operator, which includes operator's investment return rate, profit target and market competitiveness etc.

Figure 2 exhibits unrealistic nature. Here at zero tariff rate the consumer surplus is $16 \mathrm{Tk} / \mathrm{min}$, which is not possible because con-

